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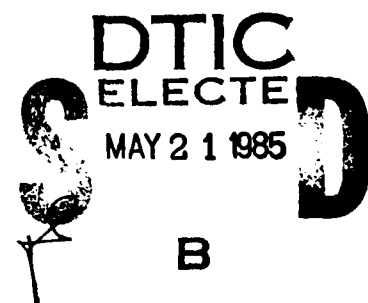# Variability of Measures of Weapons Effectiveness

Volume V: Techniques of the Methodology of Volumes I, II, III, and IV

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This report consists of a collection of the relevant equations which are developed in Volumes I, II, III, and IV of the report "Variability of Measures of Weapons Effectiveness". The equations presented are for the estimation of the probability of kill and for the variance of the probability of kill for ground targets which are either sensitive to blast or to fragmentation when delivery error is present and when it is not.

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PREFACE

This report describes work done during 1984 by Dr J. F. Mahoney, investigator, from the Department of Industrial and Systems Engineering, the University of Florida, Gainesville, Florida 32611, under Contract No. F08635-83-C-0202 sponsored by the Air Force Armament Laboratory (AFATL), Armament Division, Eglin Air Force Base, Florida 32542. The program manager was Mr Daniel A. McInnis (DLYW).

The work provides the mathematical and statistical techniques of the methodology developed in the four series of reports entitled "Variability of Measures of Weapons Effectiveness," Volumes I, II, III, and IV, in the form of equations that can be coded for high speed computers.

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SECTION I

INTRODUCTION

This report summarizes the important equations and calculations which were developed in the four-volume research report "Variability of Measures of Weapons Effectiveness" written primarily by B. S. Sivazlian, [2],[3],[4], and [5]. Section II, III, IV, and V correspond on a one-to-one basis, to Volumes I, II, III, and IV of the above mentioned report. The objective is to present equations for the estimation of probability of kill and its variance for targets which are only sensitive to blast or to fragments when there is delivery error and when there is not.

SECTION II

FRAGMENTS SENSITIVE TARGETS IN THE
ABSENCE OF DELIVERY ERROR

1. Objective

The equations necessary to compute $E[P_{kf}]$ and $Var[P_{kf}]$ in the absence of delivery error are presented. Two approaches are used.

2. Input Parameters

Numerical values of the following must be provided:

R_{x_1} = minimum estimate of the weapon radius in the direction of range, [ft].

R_{x_2} = maximum estimate of the weapon radius in direction of range, [ft].

R_{y_1} = minimum estimate of the weapon radius in the direction of deflection, [ft].

R_{y_2} = maximum estimate of the weapon radius in the direction of deflection, [ft].

x = impact coordinate in the direction of range where the target is at x=0, [ft].

y = impact coordinate in the direction of deflection where the target is at y=0, [ft].

3. Special Functions

One must have access to $P(X|Y)$ which is the chi-squared probability function. In SAS this is supplied by PROBCHI(X,Y). The related function $Q(X|Y) = 1 - P(X|Y)$ may be found in Table 26.7 of Abramowitz and Stegun [1]. For checking purposes $P(7|10) = .27456$.

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4. Defined Function

The following function is now easily computed:

$$J(A,B,k) = \frac{k\sqrt{\pi}\ \frac{[P(2\frac{k^2}{B^2}|1) - P(2\frac{k^2}{A^2}|1)]}{B-A}} {} + \frac{B\ \exp(-\frac{k^2}{B^2}) - A\ \exp(-\frac{k^2}{A^2})}{B-A}$$

This equation is a recasting of (7) of [2]. It was obtained from (7) by first dividing through by B-A and then using the substitution

$$\Phi(x) = \frac{P(x^2|1)}{2}$$

The prime benefit is that the first two equations of the next section are simpler in appearance. These two equations are exactly (8) and (10) of [2].

5. Subjective Estimation Method

$$E[P_{kf}] = \overline{J}(R_{y_1}, R_{y_2}, y)\ \overline{J}(R_{x_1}, R_{x_2}, x)$$

$$E[P_{kf}^2] = \overline{J}(R_{y_1}, R_{y_2}, y\sqrt{2})\ \overline{J}(R_{x_1}, R_{x_2}, x\sqrt{2})$$

$$Var[P_{kf}] = E[P_{kf}^2] - (E[P_{kf}])^2$$

$$\sigma_{kf} = \sqrt{Var[P_{kf}]}$$

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6. Taylor's Series Method.

First compute

$$\bar{R}_x = \frac{R_{x_2} + R_{x_1}}{2}$$

$$\bar{R}_y = \frac{R_{y_2} + R_{y_1}}{2}$$

$$Var[R_x] = \frac{(R_{x_2} - R_{x_1})^2}{12}$$

$$Var[R_y] = \frac{(R_{y_2} - R_{y_1})^2}{12}$$

Then compute

$$E[P_{kf}] = exp\left[-\left(\frac{x^2}{\bar{R}_x^2} + \frac{y^2}{\bar{R}_y^2}\right)\right]$$

$$Var[P_{kf}] = \left[\frac{2x^2 \, E[P_{kf}]}{\bar{R}_x^3}\right]^2 Var[R_x]$$

$$+ \left[\frac{2y^2 \, E[P_{kf}]}{\bar{R}_y^3}\right]^2 Var[R_y]$$

$$\sigma_{P_{kf}} = \sqrt{Var[P_{kf}]}$$

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SECTION III

BLAST SENSITIVE TARGETS IN THE
ABSENCE OF DELIVERY ERROR

1. Objective

The equations necessary to compute $E[P_{kf}]$ and $Var[P_{kf}]$ for blast

sensitive targets in the absence of delivery error are presented.

2. Input Parameters

Numerical values of the following must be provided:

A_1 = minimum estimate of the radius for which
kill is total, [ft].

A_2 = maximum estimate of the radius for which
kill is total, [ft].

B_1 = minimum estimate of the radius for which
there is no kill, [ft].

B_2 = maximum estimate of the radius for which
there is no kill, [ft].

These numbers conform to

$$0 < A_1 < A_2 < B_1 < B_2 < \infty$$

3. Annular Regions and Special Conditions

Five annualar regions are identified which are centered about the point

of impact. R is the distance in feet from impact. The regions are:

Region a: $0 < R < A_1$

Region b: $A_1 < R < A_2$

Region c: $A_2 < R < B_1$

Region d: $B_1 < R < B_2$

Region e: $B_2 < R < \infty$

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In addition, the relations among A_1, A_2, B_1, and B_2 admit to four conditions. They are

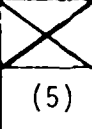Condition 1: $A_1 = A_2 \equiv A$, $B_1 = B_2 \equiv B$
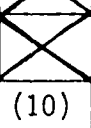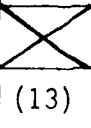
Condition 2: $A_1 < A_2$, $B_1 = B_2 \equiv B$

Condition 3: $A_1 = A_2 \equiv A$, $B_1 < B_2$

Condition 4: $A_1 < A_2$, $B_1 < B_2$

4. Subjective Estimation Method

The equations for calculating $E[P_{kf}]$, $E[P_{kf}^2]$, and $Var[P_{kf}]$ have been extracted from [3]. The table given below acts as a directory. One must first choose which region (a,b,c,d, or e) is of interest and then which condition (1,2,3, or 4) applies. Then, from the table, find the reference number which directs one to the appropriate set of equations which are presented after the table.

	Region a	Region b	Region c	Region d	Region e
Condition 1	(1)		(2)		(3)
Condition 2	(4)	(5)	(6)		(7)
Condition 3	(8)		(9)	(10)	(11)
Condition 4	(12)	(13)	(14)	(15)	(16)

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(1) $E[P_{kf}] = 1$

         $E[P_{kb}^2] = 1$

         $Var[P_{kb}] = 0$

(2) $E[P_{kb}] = \dfrac{B-R}{B-A}$

         $E[P_{kb}] = \left(\dfrac{B-R}{B-A}\right)^2$

         $Var[P_{kb}] = 0$

(3) $E[P_{kb}] = 0$

         $E[P_{kb}^2] = 0$

         $Var[P_{kb}] = 0$

(4) $E[P_{kb}] = 1$

         $E[P_{kb}^2] = 1$

         $Var[P_{kb}] = 0$

(5) $E[P_{kb}] = \dfrac{B-R}{A_2-A_1} \ln \dfrac{B-A_1}{B-R} + \dfrac{A_2-R}{A_2-A_1}$

         $E[P_{kb}^2] = \dfrac{(B-R)(R-A_1)}{(B-A_1)(A_2-A_1)} + \dfrac{A_2-R}{A_2-A_1}$

         $Var[P_{kb}] = E[P_{kb}^2] - (E[P_{kb}])^2$

(6) $E[P_{kb}] = \dfrac{B-R}{A_2-A_1} \ln \dfrac{B-A_1}{B-A_2}$

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$$E[P^2_{kb}] = \frac{(B-R)^2}{(B-A_2)(B-A_1)}$$

$$Var[P_{kb}] = E[P^2_{kb}] - (E[P_{kb}])^2$$

(7)　　$E[P_{kb}] = 0$

$$E[P^2_{kb}] = 0$$

$$Var[P_{kb}] = 0$$

(8)　　$E[P_{kb}] = 1$

$$E[P^2_{kb}] = 1$$

$$Var[P_{kb}] = 0$$

(9)　　$E[P_{kb}] = 1 - \dfrac{R-A}{B_2-B_1} \ln \dfrac{B_2-A}{B_1-A}$

$$E[P^2_{kb}] = 1 - 2\frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{B_1-A} + \frac{(R-A)^2}{(B_2-A)(B_1-A)}$$

$$Var[P_{kb}] = E[P^2_{kb}] - (E[P_{kb}])^2$$

(10)　　$E[P_{kb}] = \dfrac{B_2-R}{B_2-B_1} - \dfrac{R-A}{B_2-B_1} \ln \dfrac{B_2-A}{R-A}$

$$E[P^2_{kb}] = \frac{B_2-R}{B_2-B_1} + \frac{(R-A)(B_2-R)}{(B_2-B_1)(B_2-A)} - 2\frac{R-A}{B_2-B_1} \ln \frac{B_2-A}{R-A}$$

$$Var[P_{kb}] = E[P^2_{kb}] - (E[P_{kb}])^2$$

(11)　　$E[P_{kb}] = 0$

$$E[P^2_{kb}] = 0$$

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$$\text{Var}[P_{kb}] = 0$$

(12) $$E[P_{kb}] = 1$$

$$E[P_{kb}^2] = 1$$

$$\text{Var}[P_{kb}] = 0$$

(13) $$E[P_{kb}] = \frac{1}{2}\left[1 + \frac{A_2 - R}{A_2 - A_1}\right] + \frac{\left(\frac{1}{2}\right)}{(A_2 - A_1)(B_2 - B_1)}\left[(B_2 - R)^2 \ln \frac{B_2 - A_1}{B_2 - R}\right.$$

$$\left. - (B_1 - R)^2 \ln \frac{B_1 - A_1}{B_1 - R} - (R - A_1)^2 \ln \frac{B_2 - A_1}{B_1 - A_1}\right]$$

$$E[P_{kb}^2] = 1 - \frac{(R - A_1)^2}{(A_2 - A_1)(B_2 - B_1)} \ln \frac{B_2 - A_1}{B_1 - A_1}$$

$$\text{Var}[P_{kb}] = E[P_{kb}^2] - (E[P_{kb}])^2$$

(14) $$E[P_{kb}] = \frac{1}{2} + \frac{\left(\frac{1}{2}\right)}{(A_2 - A_1)(B_2 - B_1)}\left[(B_2 - R)^2 \ln \frac{B_2 - A_1}{B_2 - A_2} - (B_1 - R)^2 \ln \frac{B_1 - A_1}{B_1 - A_2}\right.$$

$$\left. - (R - A_1)^2 \ln \frac{B_2 - A_1}{B_1 - A_1} + (R - A_2)^2 \ln \frac{B_2 - A_2}{B_1 - A_2}\right]$$

$$E[P_{kb}^2] = 1 - \frac{(R - A_1)^2}{(A_2 - A_1)(B_2 - B_1)} \ln \frac{B_2 - A_1}{B_1 - A_1} + \frac{(R - A_2)^2}{(A_2 - A_1)(B_2 - B_1)} \ln \frac{B_2 - A_2}{B_1 - A_2}$$

$$\text{Var}[P_{kb}] = E[P_{kb}^2] - (E[P_{kb}])^2$$

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$$(15) \quad E[P_{kb}] = \frac{(B_2-R)}{2(B_2-B_1)} + \frac{(\frac{1}{2})}{(A_2-A_1)(B_2-B_1)} \left[(R-A_2)^2 \ln \frac{B_2-A_2}{R-A_2} - (R-A_1)^2 \ln \frac{B_2-A_1}{R-A_1} \right.$$

$$\left. - (B_2-R)^2 \ln \frac{B_2-A_2}{B_2-A_1} \right]$$

$$E[P_{kb}^2] = \frac{B_2-R}{B_2-B_1} + \frac{1}{(A_2-A_1)(B_2-B_1)} \left[(R-A_2)^2 \ln \frac{B_2-A_2}{R-A_2} - (R-A_1)^2 \ln \frac{B_2-A_1}{R-A_1} \right]$$

$$Var[P_{kb}] = E[P_{kb}^2] - (E[P_{kb}])^2$$

$$(16) \quad E[P_{kb}] = 0$$

$$E[P_{kb}^2] = 0$$

$$Var[P_{kb}] = 0$$

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FRAGMENT SENSITIVE TARGETS IN THE PRESENCE OF DELIVERY ERROR

1. Objective

The equations necessary to computer $E[P_{kf}]$ and $Var[P_{kf}]$ for fragment sensitive targets in the presence of delivery error are presented. Two approaches are used.

2. Input Parameters

Numerical values of the following parameters must be provided.

R_{x_1} = minimum estimate of weapons radius in the direction of range, [ft].

R_{x_2} = maximum estimate of weapon radius in the direction of range, [ft].

R_{y_1} = minimum estimate of weapon radius in the direction of deflection, [ft].

R_{y_2} = maximum estimate of weapon radius in the direction of deflection, [ft].

σ_{x_1} = minimum value of the standard deviation of the aiming error in the direction of range, [ft].

σ_{x_2} = maximum value of the standard deviation of the aiming error in the direction of range, [ft].

σ_{y_1} = minimum value of the standard deviation of the aiming error in the direction of deflection, [ft].

σ_{y_2} = maximum value of the standard deviation of the aiming error in the direction of deflection, [ft].

3. Defined Quantities

The eight quantities listed below must be calculated.

$$X_{11} = \sqrt{1 + \frac{R_{x_1}^2}{2\sigma_{x_1}^2}}$$

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$$X_{12} = \sqrt{1 + \frac{R_{x_1}^2}{2\sigma_{x_2}^2}}$$

$$X_{21} = \sqrt{1 + \frac{R_{x_2}^2}{2\sigma_{x_1}^2}}$$

$$X_{22} = \sqrt{1 + \frac{R_{x_2}^2}{2\sigma_{x_2}^2}}$$

$$Y_{11} = \sqrt{1 + \frac{R_{y_1}^2}{2\sigma_{y_1}^2}}$$

$$Y_{12} = \sqrt{1 + \frac{R_{y_1}^2}{2\sigma_{y_2}^2}}$$

$$Y_{21} = \sqrt{1 + \frac{R_{y_2}^2}{2\sigma_{y_1}^2}}$$

$$Y_{22} = \sqrt{1 + \frac{R_{y_2}^2}{2\sigma_{y_2}^2}}$$

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4. Subjective Estimation Method

Calculate the following:

$$E[I_x] = \frac{1}{\sqrt{2}\,(R_{x_2} - R_{x_1})(\sigma_{x_2} - \sigma_{x_1})} \left[\sigma_{x_2}^2 (X_{22} - X_{12}) - \sigma_{x_1}^2 (X_{21} - X_{11}) \right.$$

$$\left. + \frac{R_{x_2}^2}{2} \ln \frac{\sigma_{x_2}(1 + X_{22})}{\sigma_{x_1}(1 + X_{21})} - \frac{R_{x_1}^2}{2} \ln \frac{\sigma_{x_2}(1 + X_{12})}{\sigma_{x_1}(1 + X_{11})} \right.$$

$$E[I_y] = \frac{1}{\sqrt{2}\,(R_{y_2} - R_{y_1})(\sigma_{y_2} - \sigma_{y_1})} \left[\sigma_{y_2}^2 (Y_{22} - Y_{12}) - \sigma_{y_1}^2 (Y_{21} - Y_{11}) \right.$$

$$\left. + \frac{R_{y_2}^2}{2} \ln \frac{\sigma_{y_2}(1 + Y_{22})}{\sigma_{y_1}(1 + Y_{21})} - \frac{R_{y_1}^2}{2} \ln \frac{\sigma_{y_2}(1 + Y_{12})}{\sigma_{y_1}(1 + Y_{11})} \right]$$

$$E[P_{kf}] = E[I_x]\, E[I_y]$$

$$E[I_x^2] = \frac{1}{2} + \frac{\sqrt{2}}{(R_{x_2} - R_{x_1})(\sigma_{x_2} - \sigma_{x_1})} \left[\frac{\sigma_{x_2}^2 X_{22}^2}{2} \sin^{-1} \frac{1}{X_{22}} - \frac{\sigma_{x_1}^2 X_{21}^2}{2} \sin^{-1} \frac{1}{X_{21}} \right.$$

$$\left. - \frac{\sigma_{x_2}^2 X_{12}^2}{2} \sin^{-1} \frac{1}{X_{12}} + \frac{\sigma_{x_1}^2 X_{11}^2}{2} \sin^{-1} \frac{1}{X_{11}} \right]$$

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$$E[I_y^2] = \frac{1}{2} + \frac{\sqrt{2}}{(R_{y_2} - R_{y_1})(\sigma_{y_2} - \sigma_{y_1})} \left[\frac{\sigma_{y_2}^2 \, Y_{22}^2}{2} \sin^{-1} \frac{1}{Y_{22}} - \frac{\sigma_{y_1}^2 \, Y_{21}^2}{2} \sin^{-1} \frac{1}{Y_{21}} \right.$$

$$\left. - \frac{\sigma_{y_2}^2 \, Y_{12}^2}{2} \sin^{-1} \frac{1}{Y_{12}} + \frac{\sigma_{y_1}^2 \, Y_{11}^2}{2} \sin^{-1} \frac{1}{Y_{11}} \right]$$

$$Var[P_{kf}] = E[I_x^2] \, E[I_y^2] - (E[P_{kf}])^2$$

The equations for $E[I_x^2]$ and $E[I_y^2]$ that are given above are mere
rearrangements of (23) and (24) of [4]. In the versions just presented,
greater use of the X_{ij} and Y_{ij} has been made and also identities of the form

$$\tan^{-1} \frac{\sigma_{x_i} \sqrt{2}}{R_{x_j}} = \tan^{-1} \frac{1}{\sqrt{X_{ji}^2 - 1}} = \sin^{-1} \frac{1}{X_{ji}}$$

have been incorporated.

5. Taylor's Series Method

Estimates of $E[P_{kf}]$ and $Var]P_{kf}]$ may be computed using a Taylor's series
approach. First calculate,

$$\overline{R}_x = \frac{R_{x_2} + R_{x_1}}{2}$$

$$\overline{R}_y = \frac{R_{y_2} + R_{y_1}}{2}$$

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$$\overline{\sigma}_x = \frac{\sigma_{x_2} + \sigma_{x_1}}{2}$$

$$\overline{\sigma}_y = \frac{\sigma_{y_2} + \sigma_{y_1}}{2}$$

$$Var[R_x] = \frac{(R_{x_2} - R_{x_1})^2}{12}$$

$$Var[R_y] = \frac{(R_{y_2} - R_{y_1})^2}{12}$$

$$Var[\sigma_x] = \frac{(\sigma_{x_2} - \sigma_{x_1})^2}{12}$$

$$Var[\sigma_y] = \frac{(\sigma_{y_2} - \sigma_{y_1})^2}{12}$$

Then compute

$$E[P_{kf}] = \frac{\overline{R}_x}{\sqrt{\overline{R}_x^2 + 2\overline{\sigma}_x^2}} \cdot \frac{\overline{R}_x}{\sqrt{\overline{R}_y^2 + 2\overline{\sigma}_y^2}}$$

and

$$Var[P_{kf}] = \left[\frac{E[P_{kf}]}{1 + \frac{\overline{R}_x^2}{2\overline{\sigma}_x^2}} \right]^2 \left[\frac{Var[R_x]}{\overline{R}_x^2} + \frac{Var[\sigma_x]}{\overline{\sigma}_x^2} \right]$$

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$$+\left[\frac{E[P_{kf}]}{1+\dfrac{\overline{R}_y^2}{2\overline{\sigma}_y^2}}\right]^2\left[\frac{\mathrm{Var}[R_y]}{\overline{R}_y^2}+\frac{\mathrm{Var}[\sigma_y]}{\overline{\sigma}_y^2}\right]$$

This equation is an algebraic rearrangement of (48) of [4].

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SECTION V

BLAST SENSITIVE TARGET IN THE PRESENCE
OF DELIVERY ERROR

1. __Objective__

The equations necessary to compute $E[P_{kb}]$ and $Var[P_{kb}]$ for blast

sensitive targets in the presence of delivery error are presented.

2. __Input Parameters__

Numerical values of the following must be provided.

A_1 = minimum estimate of the radius for which
kill due to blast in complete, [ft].

A_2 = maximum estimate of the radius for which
kill due to blast is complete, [ft].

B_1 = minimum estimate of the radius for which there
is no kill due to blast, [ft].

B_2 = maximum estimate of the radius for which there
is no kill due to blast, [ft].

σ_{x_1} = minimum value of the standard deviation of the aiming error
in the direction of range, [ft].

σ_{x_2} = maximum value of the standard deviation of the aiming error
in the direction of range, [ft].

σ_{y_1} = minimum value of the standard deviation of the aiming error
in the direction of deflection, [ft].

σ_{y_2} = maximum value of the standard deviation of the aiming error
in the direction of deflection, [ft].

3. Defined Quantities

The following quantities are easily computed

$$\bar{A} = E[A] = \frac{A_2 + A_1}{2}$$

$$\bar{B} = E[B] = \frac{B_2 + B_1}{2}$$

$$\bar{\sigma}_x = E[\sigma_x] = \frac{\sigma_{x_2} + \sigma_{x_1}}{2}$$

$$\bar{\sigma}_y = E[\sigma_y] = \frac{\sigma_{y_2} + \sigma_{y_1}}{2}$$

$$Var[A] = \frac{(A_2 - A_1)^2}{12}$$

$$Var[B] = \frac{(B_2 - B_1)^2}{12}$$

$$Var[\sigma_x] = \frac{(\sigma_{x_2} - \sigma_{x_1})^2}{12}$$

$$Var[\sigma_y] = \frac{(\sigma_{y_2} - \sigma_{y_1})^2}{12}$$

$$a = \frac{1}{4}\left(\frac{1}{\bar{\sigma}_x^2} + \frac{1}{\bar{\sigma}_y^2}\right)$$

$$b = \frac{1}{4}\left(\frac{1}{\bar{\sigma}_y^2} - \frac{1}{\bar{\sigma}_x^2}\right)$$

$$\alpha = 2a\bar{A}^2 \qquad\qquad \xi = \left(\frac{b}{2a}\right)^2$$

$$\beta = 2a\bar{B}^2 \qquad\qquad \eta = \left(\frac{b}{8a}\right)^2$$

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4. Special Function

One must have access to $P(X|Y)$ which is the chi-squared probability function. In SAS this is supplied by PROBCHI(X,Y). The related function $Q(X|Y) = 1 - P(X|Y)$ may be found in Table 26.7 of Abramowitz and Stegun [1]. For checking purposes $P(7|10) = .27456$.

5. Additional Defined Quantities

The quantities $X_1(\overline{A})$, $X_1(\overline{B})$, $X_2(\overline{A})$, $X_2(\overline{B})$, $Y_2(\overline{A})$ and $Y_2(\overline{B})$ are all given by infinite series which are of the form

$$C[t_1 + t_2 + ...+ t_n + ...]$$

The definitions of the entering items in this series differ depending on which of the six quantities is to be calculated. The series all converge rapidly so that only the first few terms of the series need be generated.

(a) Calculation of $X_1(\overline{A})$.

$$C = \frac{1}{2a}$$

$$t_n = P(\alpha|4n-2) \, \xi^{n-1} \, \frac{(2n-2)!}{(n-1)! \, (n-1)!} \, , \quad n=1,2,...$$

$$X_1(\overline{A}) = C[t_1 + t_2 + ...+ t_N]$$

where N is the number of terms retained in the series. N may be taken as the smallest integer for which

$$\frac{t_N}{t_1} < 10^{-6}$$

(b) Calculation of $X_1(\overline{B})$

$$C = \frac{1}{2a}$$

$$t_n = P(\beta|4n-2)\; \xi^{n-1}\; \frac{(2n-1)!}{(n-1)!\;(n-1)!}\;,\quad n=1,2,\ldots$$

$$X_1(\overline{B}) = C[t_1 + t_2 + \ldots + t_N]$$

where N is the smallest integer for which

$$\frac{t_N}{t_1} < 10^{-6}$$

(c) Calculation of $X_2(\overline{A})$

$$C = \frac{\sqrt{\pi}}{4a^{3/2}}$$

$$t_n = P(\alpha|4n-1)n^{n-1}\; \frac{(4n-3)!}{(n-1)!\;(n-1)!\;(2n-2)!}\;,\quad n=1,2,\ldots$$

$$X_2(\overline{A}) = C[t_1 + t_2 + \ldots + t_N]$$

where N is the smallest integer for which

$$\frac{t_N}{t_1} < 10^{-6}$$

(d) Calculation of $X_2(\overline{B})$

$$C = \frac{\sqrt{\pi}}{4a^{3/2}}$$

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$$t_n = P(\beta|4n-1)n^{n-1} \frac{(4n-3)!}{(n-1)! \, (n-1)! \, (2n-2)!} \; , \quad n=1,2,\ldots$$

$$X_2(\overline{B}) = C[t_1 + t_2 + \ldots + t_N]$$

where N is the smallest integer for which

$$\frac{t_N}{t_1} < 10^{-6}$$

(e) Calculation of $Y_2(\overline{A})$.

$$C = \frac{b\sqrt{\pi}}{32 \, a^{5/2}}$$

$$t_n = P(\alpha|4n+1)n^{n-1} \frac{(4n-1)!}{n! \, (n-1)! \, (2n-1)!} \; , \quad n = 1,2,\ldots$$

$$Y_2(\overline{A}) = C[t_1 + t_2 + \ldots + t_N]$$

where N is the smallest integer for which

$$\frac{t_N}{t_1} < 10^{-6}$$

(f) Calculation of $Y_2(\overline{B})$

$$C = \frac{b\sqrt{\pi}}{32a^{5/2}}$$

$$t_n = P(\beta|4n+1)n^{n-1} \frac{(4n-1)!}{n! \, (n-1)! \, (2n-1)!} , \quad n = 1,2,\ldots$$

$$Y_2(\overline{B}) = C[t_1 + t_2 + \ldots + t_N]$$

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where N is the smallest integer for which

$$\frac{t_N}{t_1} < 10^{-6}$$

Items (a) through (f) given above are rearrangements of (54) and (55) of [5] for particular parameter values.

6. Taylor's Series Method

$$E[P_{kb}] = \frac{\overline{B}X_1(\overline{B}) - \overline{A}X_1(\overline{A}) - X_2(\overline{B}) + X_2(\overline{A})}{(\overline{B}-\overline{A})\overline{\sigma}_x\overline{\sigma}_y}$$

Then calculate the quantities

$$R_1 = \frac{E[P_{kb}]}{\overline{B}-\overline{A}} - \frac{X_1(\overline{A})}{\overline{\sigma}_x\overline{\sigma}_y(\overline{B}-\overline{A})}$$

$$R_2 = \frac{X_1(\overline{B})}{\overline{\sigma}_x\overline{\sigma}_y(\overline{B}-\overline{A})} - \frac{E[P_{kb}]}{\overline{B}-\overline{A}}$$

$$R_3 = - \frac{X_2(\overline{B}) - X_2(\overline{A}) + Y_2(\overline{B}) - Y_2(\overline{A})}{2\overline{\sigma}_x^2\overline{\sigma}_y(\overline{B}-\overline{A})}$$

$$R_4 = \frac{Y_2(\overline{B}) - Y_2(\overline{A}) - X_2(\overline{B}) + X_2(\overline{A})}{2\overline{\sigma}_x\overline{\sigma}_y^2(\overline{B}-\overline{A})}$$

$$Var[P_{kb}] = R_1^2 \, Var[A] + R_2^2 \, Var[B]$$

$$+ R_3^2 \, Var[\sigma_x] + R_4^2 \, Var[\sigma_y]$$

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[5] Sivazlian, B.D., and J.F. Mahoney, Variability of Measures of Weapons Effectiveness, Volume IV: Application to Blast Sensitive Targets in the Presence of Delivery Error, Air Force Armament Laboratory, AFATL/DLYW, Eglin Air Force Base, FL 32542, November 1984.

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